Examinee Info. 学生资料

Country/国 家: ________ City/市（省）: ________ School/学 校: ________

Name/姓 名: ________ Sex/性 别: ________ Instructor/辅导老师: ________

Senior High School First Year Contest Problem

Examination Time：90 min Total Point：100points Score：________

- Contestant must write down the answer of each problem in the blank, answer with erasure will not be credited!
- For Problem 17 and 18, presentation of solution on the space provided is a must, no credit will be given if only the final answer is written down in the paper!

Multiple Choice	1	2	3	4	5	6	7	8
Answer								
Fill-in the blank	9	10	11	12	13	14	15	16
Answer								

A. Multiple-Choice Problems.（5 points each, a total of 40 points）

1. What are the possible values of a such that $A\subseteq(A\cap B)$, where both $A=\{x\mid 2a+1\le x\le 3a-5\}$ and $B=\{x\mid 3\le x\le 22\}$ are non-empty set?

   A. $\{a\mid 6\le a\le 9\}$ B. $\{a\mid a\le 9\}$ C. $\{a\mid 1\le a\le 9\}$ D. $\varnothing$

2. If the function $y=f^{-1}(x)$ is the inverse function of $y=f(x)$ and the graph of $y=f^{-1}(x+5)$ passes through the point $(-1，3)$，then the graph of $y=f(x+1)$ will pass through what following point of coordinate?

   A. (2,4) B. (3,–1) C. (–2,8) D. (3,4)

3. Determine the monotonic range of $f(x)=\log_{\frac{1}{2}}\cos x$ where $x\in(0,2\pi)$.

   A. $(\pi,2\pi)$ B. $(0,\pi)$ C. $\left(0,\frac{\pi}{2}\right)$ D. $\left(\frac{3\pi}{2},2\pi\right)$

4. Select any three distinct numbers from $\{1, 2, 3, \cdots, 20\}$ and hence, when these three numbers are arranged in arithmetic sequence of ascending order. How many possible arithmetic sequence are there?

   A. 90 B. 120 C. 180 D. 240

5. The three vertices of $\triangle ABC$ are A, B, C and P is a point the plane such that $\overline{PA}+\overline{PB}+\overline{PC}=\overline{AB}$.，Determine the relationship of point P and $\triangle ABC$.

   A. P is on the straight line of side AB B. P is an exterior point of $\triangle ABC$

   C. P is an interior point of $\triangle ABC$ D. P is a trisection point on side

6. If the terminal side of θ is in Second Quadrant and $\sin\frac{\theta}{2}<\cos\frac{\theta}{2}$, then what is the value of $2^{\left|\log_2\left|\cos\frac{\theta}{2}\right|\right|}$?

   A. $\sec(\pi-\frac{\theta}{2})$ B. $\cos\frac{\theta}{2}$ C. $\sec(-\frac{\theta}{2})$ D. $-\cos(\frac{\theta}{2}-\pi)$

7. Mary randomly selected 3 digits from 1 to 9 and then arranged these as a 3-digit number with the bigger digit in the higher place value. Nestor also randomly selected 3 digits from 1 to 8 and followed the procedure as given above to form another 3-digit number. What is the probability that the 3-digit number of Mary is selected is greater than the 3-digit number of Nestor selected?

   A. $\frac{47}{72}$ B. $\frac{49}{72}$ C. $\frac{37}{56}$ D. $\frac{2}{3}$

8. How many elements are there in a set $A=\left\{(x,y,z)\,\middle|\,x^2+y^2+z^2+4=xy+3y+2z, x,y,z\in R\right\}$?

   A. 1 B. 2 C. 0 D. None

B. Fill in the blank.（5 points each, a total of 40 points）

9. Let $f(x)=(4a-3)x+b-2a$, $x\in[0,1]$ and $f(0)\le 2$, $f(1)\le 2$. What is the maximum value of $a+b$?

10. For any real numbers x and y, we have a function $f(x)$ satisfy $f(x+y)=f(x)f(y)$ and $f(1)=2$. What is the numerical value of $\frac{f(2)}{f(1)}+\frac{f(3)}{f(2)}+\cdots\frac{f(2005)}{f(2004)}$?

11. If A and B are acute angles that satisfy $\tan A\cdot\tan B=\tan A+\tan B+1$, then what is the value of $\cos(A+B)$?

12. Given two sets: $A=\{(x,y)\mid y=ax+2\}$, $B=\{(x,y)\mid y=|x+1|\}$ and cardinal$(A\cap B)=1$, what are the possible values of a?

13. Give a prime factors of $1+2^{21}+4^{21}$.

14. Let real numbers x, y satisfy the system of equations $\begin{cases}(x-1)^3+2005(x-1)=-1,\\(y-1)^3+2005(y-1)=1.\end{cases}$

What is the numerical value of $x+y$?

15. Procedure of performing a *bubble sort* operation to a series $a_1, a_2, \cdots, a_n$ is as follows: First, compare the size of a1 from the first and a2 from the second terms in the given series. If $a_1 > a_2$, then swap the two positions in the series (that is, exchange their value), or remain unchanged if they are the equal; then compare the size of second and third term in the series, follow the same rule, that is; if $a_2 > a_3$, then swap, or else remain unchanged if they are equal; then compare the size of the third and fourth and so on , $\cdots$, according to the same rules, when reaching the last two terms, after compare the size of a_{n-1}, a_n and after the end of swapping the location, then stop. Now randomly arrange $1, 2, 3, \cdots, 2013$ as a series and perform the above described "Bubble Sort" technique to this series of 2013 terms. What is the probability that after bubble sort the number in the 10th term is the number in the 5th term?

16. Let $f(x)=\left|\left|\cdots\left|x^{10}-2^{2013}\right|-2^{2012}\right|-\cdots-2^2\right|-2\right|$. What is the value of $f(2013)$?

C. Problem Solving. (10 points each, a total of 20 points. Show your brief solution on the space below each question)

17. Given: $A\subseteq\{1,2,3,\cdots\cdots,2013\}$. What is the largest possible number of pairs of elements in set A so that the absolute value of the difference of any two elements is neither equal to 3 nor 4 nor 5?

18. The triplet (A, B, C)can be transformed with the following two steps:

Transformation 1: Three numbers can be rearranged arbitrarily

Transformation 2: Convert (A, B, C) into ($2B+2C-A$, B, C);

Assume that the initial state of a given triplet is (−1, 0, 1).

(a) Can we perform a finite number of steps of transformation to obtain a triplet as (2012, 2013, 2014). Explain .

(b) Can we perform a finite number of steps of transformation to obtain a triplet such as (2009, 2010, 2011). Explain.

(c) Determine all possible values of x so that there is a finite number of steps of transformation to obtain a triplet as (1, 2024, x). Explain