Examinee Info. 学生资料

Country/国家:____ City/市（省）:____ School/学校:____

Name/姓名:____ Sex/性别:____ Instructor/辅导老师:____

2013年第九屆“IMC國際數學競賽”(新加坡)
Ninth IMC International Mathematics Contest (Singapore),2013

Primary Six Contest Problem

Examination Time：90 min Total Point：100points Score：_____

- Contestant must write down the answer of each problem in the blank, answer with erasure will not be credited!
- For Problem 17 and 18, presentation of solution on the space provided is a must, no credit will be given if only the final answer is written down in the paper!

Multiple Choice	1	2	3	4	5	6	7	8
Answer								
Fill-in the blank	9	10	11	12	13	14	15	16
Answer								

A. Multiple-Choice Problems.（5 points each, a total of 40 points）

1. What is the sum of all the first 2013 digits after the decimal point when the result of $\frac{1001}{1443}\times\frac{10101}{14443}$ is written in repeating decimal form?

   A. 9062 B. 9059 C. 9058 D. 9055

2. Determine the sum of all the digits in the final product of $9\times99\times9999\times\underbrace{99\cdots9}_{(8)}\times\underbrace{99\cdots9}_{(16)}$.

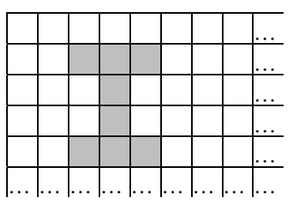
   A. 144 B. 120 C. 96 D. 72

3. Find a 7-digit number $\overline{ABCDCBA}$ such that each letter can only represent one digit, the same letter stands for the same digit. It is also given that the first digit of this 7-digit number is divisible by 2, the first two digits is divisible by 3, the first three digits is divisible by 4, …, the first seven digits is divisible by 8. What does $\overline{ABCDCBA}$ represent?

   A. 2785872 B. 4285824 C. 6925296 D. 8425248

4. Write one natural number in each grid starting from 1 begin from the left moving to right so that there are *m* natural numbers in every row, and all these numbers form a rectangular shape. Now, use a frame of I-shape to cover eight numbers such that the sum of these eight numbers is 312. What is the possible value of *m*?

   A. 6 B. 13 C. 20 D. 30



The position of “I” in the figure is unfixed, only for example.

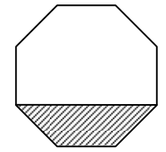
5. A 100 kg barrel of salt solution whose concentration is 10%, repeat the following procedure a number of times such that the every time poured10 kg of solution out and replaced it by 10 kg salt. When will be the first time the concentration of salt solution is greater than 50%?

   A. 4 times B. 5 times C.6 times D.7 times

6. If the representation of a positive integer *N* in base 8 is 12345654321, then what is the remainder when *N* is divided by 7?

   A. 0 B. 1 C. 2 D. 3

7. Select four vertices on a regular octagon as shown at the right. Connect them to form a trapezoid (shaded region). The area of the shaded region is what part that of area of regular octagon?



   A. $\frac{1}{3}$ B. $\frac{1}{4}$ C. $\frac{2}{5}$ D. $\frac{3}{8}$

8. Using one leg (whose length is 1 unit) of a right-angled triangle to be the axis of a coin. Rotate the given triangle in one complete round to form a cone such that the lateral area of the cone is twice the area the circular base. Suppose we use the other leg of the given right-angled triangle to be the axis and rotate the triangle along this axis, then a new cone will form. What is the volume of this cylinder?
   Note: The **axis** of the cone is the line segment whose end points are the vertex and the center of the base.

   A. $\frac{\pi}{2}$ B. π C. 2π D. 3π

B. Fill in the blank.（5 points each, a total of 40 points）

9. What is the simplified value of $\frac{1\times3\times5}{2\times4}-\frac{3\times5\times7}{4\times6}+\frac{5\times7\times9}{6\times8}-\cdots+\cdots-\frac{15\times17\times19}{16\times18}+\frac{17\times19\times21}{18\times20}$?

10. Peter, Richard and Terry are traveling on their own destination and traveled at the same time. Peter starts traveling from town A and headed towards town B, while Richard and Terry left town B toward town A. 4 hours later, when Peter and Richard meet each other, after 2 hours Peter will meet Terry, then Terry continued moving forwards town A and will meet Richard at a gasoline station which is 80 km from town A. If the speed of Peter and Terry are the same, then what is the distance between towns *A* and *B*?

11. Construct two 5-digit numbers from the ten digits 0 to 9 such that each digit will use only once and one 5-digit number is nine times the other 5-digit number. What is the maximum sum of these two 5-digit numbers?

12. There are two candles of the same length with different thickness, candle *A* consumes in 3 hours while candle *B* in 2 hours. During a city electric power failure, Betty lit both candles, later when electric power resumed, it was found out the length of candle *A* remain (not yet burn) twice than that of candle *B*. Tonight, the city encounter electric power failure, so Betty continues to light the two candles, when electric power resume, it was found that the length of candle *A* is four times than that of candle *B*. How many hours was the electric power failure last tonight?

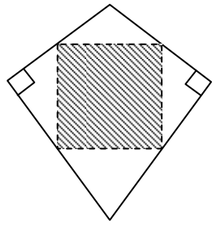
13. A Palindrome number is a number that remains the same when its digits are reversed such as 16461. If the sum of two 4-digit palindrome numbers is a 5-digit palindrome number, for example $2882 + 9339 = 12221,$ such equation is one set of palindromic equation. How many possible sets of palindromic equations will there be that will satisfy the above properties?

14. There are two three-digit numbers $\overline{abc}$ and $\overline{def}$ satisfy $\overline{abc} - \overline{def} = a + b + c + d + e + f$,

    where same letter stands for the same digit while different letters represent distinct digits.

    What is the least possible 3-digit number $\overline{def}$?

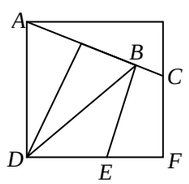
15. Given a quadrilateral figure with two right-angled which are opposite to each other and whose length of two upper adjacent sides are equal and same situation on the two lower adjacent sides. The ratio of the lengths of two unequal sides is 3:4. Now, select one point on each side of the given quadrilateral and connect them to form a square, if the side of the square and quadrilateral are both in integers. What is the least possible perimeter of the given quadrilateral in cm?



16. If there are two or more consecutive positive integers whose sum is 2013, then at most how many zeros are there in the product of these groups of consecutive positive integers?

C. Problem Solving. (10 points each, a total of 20 points. Show your detailed solution on the space below each question)

17. The square at the right is divided into five regions of equal area by 4 line segments. Determine (a) *AB*:*BC*. (b) *DE*:*EF*.



18. For a group of numbers that were arranged in one line, let us perform a certain operation to any four neighboring numbers such as *a*, *b*, *c*, *d* by re-arrange it as *d*, *c*, *b*, *a*; such kind of operation is called *a transformation*.
For a series of numbers 1, 2, 3, ...,19, 20;
(a) At least after how many transformation will the number 20 become the first term? List down the step.

(b) Is it possible that the original series of numbers be transform as 20, 13, 1, 2, 3, ...,12, 14, 15, ..., 19? If possible, list down the step or else explain why it is not possible.

新加坡 . 决赛 六年级
Singapore.Final Grade 6