Examinee Info. 学生资料

Country/国家: _____ City/市（省）: _____ School/学校: _____

Name/姓名: _____ Sex/性别: _____ Instructor/辅导老师: _____

2013年第九届“IMC國際數學競賽”(新加坡)

Ninth IMC International Mathematics Contest (Singapore),2013

Junior High School First Year Contest Problem

Examination Time：90 min　Total Point：100points　Score：

- Contestant must write down the answer of each problem in the blank, answer with erasure will not be credited!
- For Problem 17 and 18, presentation of solution on the space provided is a must, no credit will be given if only the final answer is written down in the paper!

Multiple Choice	1	2	3	4	5	6	7	8
Answer								
Fill-in the blank	9	10	11	12	13	14	15	16
Answer								

A. Multiple-Choice Problems.（5 points each, a total of 40 points）

1. What is the simplified value of $\frac{2^{n+4}-2^{n+1}}{2^{n+4}}$?

   A. $2^{n+1}-\frac{1}{8}$　　B. -2^{n+1}　　C. $\frac{7}{8}$　　D. $\frac{7}{4}$

2. If $b>a>0$，$a^2+b^2=4ab$，then what is the numerical value of $\left(\frac{a+b}{a-b}\right)^2$?

   A. 2　　B. 3　　C. 2 or 3　　D. cannot be determined

3. If $ac<0$, then how many of the following inequalities $\frac{a}{c}<0$，$ac^2<0$，$a^2c<0$，$c^3a<0$，$ca^3<0$ will satisfy the given condition?

   A. 1 inequality　B. 2 inequalities　C. 3 inequalities　D. 4 inequalities

4. Given that $25^x=2000$ and $80^y=2000$, then what is the numerical value of $\frac{1}{x}+\frac{1}{y}$?

   A. 2　　B. 1　　C. $\frac{1}{2}$　　D. $\frac{3}{2}$

5. Let *a, b, c* be positive numbers. If $x=\frac{a}{b+c}=\frac{b}{c+a}=\frac{c}{a+b}$, then determine the value of *x*.

   A. $\frac{1}{2}$　　B. 1　　C. $\frac{3}{2}$　　D. 2

6. If $|x|+||x|-1|=1$, then which of the following statement is true?

   A. $(x+1)(x-1)>0$　B. $(x+1)(x-1)<0$　C. $(x+1)(x-1)\geq 0$　　D. $(x+1)(x-1)\leq 0$

7. Let $S=\frac{2}{1\times 3}+\frac{2^2}{3\times 5}+\frac{2^3}{5\times 7}+\cdots+\frac{2^{49}}{97\times 99}$，$T=\frac{1}{3}+\frac{2}{5}+\frac{2^2}{7}+\cdots+\frac{2^{48}}{99}$，then find the value of $S-T$.

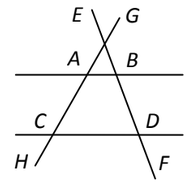
   A. $\frac{2^{49}}{99}$　　B. $1-\frac{2^{49}}{99}$　　C. $\frac{2^{49}}{99}-1$　　D. $\frac{2^{49}}{99}+1$

8. If *a, c, d* are integers and *b* is a positive integer that satisfy *a+b=c, b+c=d, c+d=a*, then what is the maximum value of *a+b+c+d*?

   A.–5　　B.–1　　C. 0　　D. 1

B. Fill in the blank.（5 points each, a total of 40 points）

9. In the given figure at the right, there are two straight line segments *AB,CD* intersect with two line segments $\overline{EF}$, $\overline{GH}$. How many pairs of interior angles on the same side of the transversal are therein the figure?



10. For all real numbers *x* and *y*, let us define an operator Δ with *x* and *y* as : $x\,\Delta\, y = ax + by + c$, where a, b, c are constants. Equation to the right is ordinary addition and multiplication. What is the numerical value of 1Δ1 when 3 Δ5=15 and 4Δ7=28?

11. Let four distinct positive integers *m*, *n*, *p* and *q* satisfy $(7-m)(7-n)(7-p)(7-q)=4$. Determine the sum of *m*, *n*, *p* and *q*.

12. If the solution of inequality $(a+b)x+(2a-3b)<0$ is $x>-\frac{1}{3}$, then what is the solution set of $(a-3b)x+(b-2a)>0$?

13. If 1 angle will separate a plane into 2 parts, then 3 angles will separate a plane at most into how many parts?

14. Given: $|a|=1, |b|=2, |c|=3$ and $a>b>c$. What is the numerical value of $(a+b-c)^2$?

15. Jerry left Town *A* for Town *B* at the same time when Larry left Town *B* for Town *A*. Both traveling at their own uniform speed. The two boys will meet after 8 hours they started to walk. If the two boys walk 2 km per hour more, both of them will met each other after 6 hours at a certain gasoline station which is 3 km from the center of Town *A* and *B*. If Jerry travels faster than Larry, then what was the speed of Jerry?

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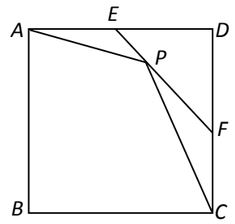
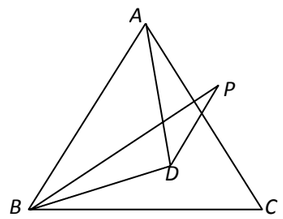
16. In square *ABCD*, *E*、*F* is the midpoint of $\overline{AD}$, $\overline{DC}$; respectively. Connect *E* and *F* such that *P* lies on $\overline{EF}$. If the area of *ABCD* is 20 cm^2, then what is the area of *ABCP* in cm^2 ?



C. Problem Solving. (10 points each, a total of 20 points. Show your detailed solution on the space below each question)

17. Refer the figure below, *D* is an interior point of equilateral$\triangle ABC$ with *DB* = *DA*, *P* is an exterior point of $\triangle ABC$ and $BP = BA$, $\angle DBP = \angle DBC$. What is the size of $\angle BPD$?



18. Let x_1, x_2, x_3, $\cdots$, x_7 be counting numbers and $x_1 < x_2 < x_3 < \cdots < x_7$ with $x_1 + x_2 + x_3 + \cdots + x_7 = 159$. Determine the maximum value of $x_1 + x_2 + x_3$.

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