Examinee Info. 学生资料

Country/国 家: ________ City/市（省）: ________ School/学 校: ________

Name/姓 名: ________ Sex/性 别: ________ Instructor/辅导老师: ________

2013年第九屆“IMC國際數學競賽”(新加坡)

Ninth IMC International Mathematics Contest (Singapore), 2013

Junior High School Second Year Contest Problem

Examination Time：90 min Total Point：100points Score：________

- Contestant must write down the answer of each problem in the blank, answer with erasure will not be credited!
- For Problem 17 and 18, presentation of solution on the space provided is a must, no credit will be given if only the final answer is written down in the paper!

Multiple Choice	1	2	3	4	5	6	7	8
Answer								
Fill-in the blank	9	10	11	12	13	14	15	16
Answer								

A. Multiple-Choice Problems.（5 points each, a total of 40 points）

1. Let $a+\frac{1}{a}=3$. What is the numerical value of $a^2+\frac{1}{a^2}$?

   A. 1 B. 7 C. 9 D. 11

2. Which of the following is the Condition for two triangles to be congruent?

   A. one angle and one side of an isosceles triangle are equal to one angle and one side of another isosceles triangle

   B. two sides of one right-angled triangle are equal to two sides of another right-angled triangle

   C. area of two triangles are equal

   D. two angles and one side of a triangle are equal to two angles and one side of another triangle

3. If a, b, c are three distinct real numbers, A, B, C are three distinct points with coordinates $A(b+c, a)$, $B(c+a, b)$, $C(a+b, c)$, then the location of these three coordinates will form what kind of relation?

   A. they will form an obtuse triangle B. they will form an acute triangle

   C. they will form a right-angled triangle D. they does not constitute a triangle

4. Straight line $y = 2x - 1$ and $y = x - k$ intersect at a point which is located on the fourth quadrant. What is the possible value of k?

   A. $k < 0.5$ B. $k > 1$ C. $0.5 < k < 1$ D. None of the above

5. How many possible values of n such that each interior angle of a regular n-side polygon is an integer?

   A.10 B. 20 C. 21 D. 22

6. How many four-digit number such as $\overline{aabb}$（where each letter can only represent one digit）is a perfect square number are there?

   A.0 B. 1 C. 2 D. 3

7. A square was partitioned into four convex polygon sides without any portion overlapping. Assume the number of sides of these four convex polygon as a, b, c, d respectively such that $a > b > c > d \geq 3$. Which of the following 4-tuple positive integers (a, b, c, d) are the possible solution sets?

   ① (6,5,4,3) ② (7,6,5,4) ③ (7,6,4,3) ④ (7,5,4,3)

   A. ①; ② B. ②; ③ C. ①; ④ D. ③; ④

8. Given: $N = 1 + 2^{15} + 4^{15}$. Which of the following is a prime factor of N?

   A.13 B.43 C.53 D.73

B. Fill in the blank.（5 points each, a total of 40 points）

9. Find a linear function whose $y = kx + b$ whose graph passes through the point $P(3,2)$ together with the straight line $x + 3y - 9 = 0$ and x-axis x will bounded an isosceles triangle.

10. Let real number a, b satisfy $a^3 + b^3 + 3ab = 1$. What are the possible sum of a and b?（Write all the possible values.）

11. Let [a] denote the greatest integer not exceeding a. For example, [4.1] = 4，[-7.2] = - 8. Determine the rational number x (express the answer as improper fraction) that satisfy the equation $x+\frac{2013}{x}=[x]+\frac{2013}{[x]}$.

12. If real numbers a、b satisfy $\sqrt{a^2-2a+1}+\sqrt{36-12a+a^2}=10-|b+3|-|b-2|$, then what is the maximum value of a^2+b^2?

13. Let a, b, c, d be four distinct real numbers that satisfy $(a + c)(a + d) = (b + c)(b + d) = 1$. Determine the value of $(a + c)(b + c)$.

14. How many solutions sets of (x, y, z) are there in the system of equations? $\begin{cases} x^2 - y^2 = 3(xz + yz + x + y) \\ y^2 - z^2 = 3(yx + zx + y + z) \\ z^2 - x^2 = 3(zy + xy + z + x) \end{cases}$

15. Procedure of performing a ***bubble sort*** operation to a series $a_1, a_2, \cdots, a_n$ is as follow: First, compare the size of first and second term in the give series. If $a_1 > a_2$, then swap the two positions in the series (that is, exchange their value), or remain unchanged; then compare the size of second and third term in the series, follow the same rule, that is; if $a_2 > a_3$, then swap, or else remain unchanged; then compare the size of the third and fourth and so on, $\cdots$, according to the same rules, when reaching the last two terms, after compare the size of a_{n-1}, a_n and after the end of swapping the location, then stop. Now randomly arrange $1, 2, 3, \cdots, 10000$ as a series with 10000 terms . What is the probability that after bubble sort the number in the 5000th term is the number of 2013 term?

16. Let function $f(x) = \left|\left|\cdots\left|x^{10} - 2^{2013}\right| - 2^{2012}\right| - \cdots - 2^2\right| - 2\right|$. What is the value of $f(2013)$?

C. Problem Solving. (10 points each, a total of 20 points. Show your brief solution on the space below each question)

17. Mary randomly select 3 digits from 1 to 9 and then arrange these as a 3-digit number with the bigger digit in the higher place value. Nestor also randomly select 3 digits from 1 to 8 and follow the procedure as above to form another 3-digit number. What is the probability that the 3-digit number of Mary is greater than the 3-digit number of Nestor ?

18. In isosceles $\triangle ABC$. $BA = BC$, $\angle ABC = 80°$, P is an interior point of the given triangle such that $\angle PAC = 10°$, $\angle PCA = 20°$. Prove that : $\angle ABP = 60°$.