Examinee Info. 学生资料

Country/国 家: ________ City/市（省）: ________ School/学 校: ________

Name/姓 名: ________ Sex/性 别: ________ Instructor/辅导老师: ________

2013年第九屆“IMC國際數學競賽”(新加坡)
Ninth IMC International Mathematics Contest (Singapore), 2013

Junior High School Third Year Contest Problem

Examination Time：90 min Total Point：100points Score：________

- Contestant must write down the answer of each problem in the blank, answer with erasure will not be credited!
- For Problem 17 and 18, presentation of solution on the space provided is a must, no credit will be given if only the final answer is written down in the paper!

Multiple Choice	1	2	3	4	5	6	7	8
Answer								
Fill-in the blank	9	10	11	12	13	14	15	16
Answer								

A. Multiple-Choice Problems.（5 points each, a total of 40 points）

1. For any value of *x*, what kind of integer is the value of algebraic expression $x^2+8x+17$?

   A.Negative numbers B. Positive numbers C. Zero D. Uncertain

2. Which of the following is the Condition for two triangles to be congruent?

   A. one angle and one side of an isosceles triangle are equal to one angle and one side of another isosceles triangle

   B. two sides of one right-angled triangle are equal to two sides of another right-angled triangle

   C. area of two triangles are equal

   D. two angles and one side of a triangle are equal to two angles and one side of another triangle

3. Let the length of three sides of $\triangle ABC$ represented as *a*, *b*, *c* satisfy the condition $2b = a + c$ and the length of three altitudes to three sides denoted as h_a, h_b, h_c. Which of the following represents the relationship of those three altitudes?

   A. $2h_b = h_a + h_c$ B. $\frac{2}{h_b}=\frac{1}{h_a}+\frac{1}{h_c}$ C. $\frac{h_b}{h_a}=\frac{h_c}{h_b}$ D. None of the above

4. Determine maximum value of *n* such that each interior angle of a *n*-sides convex polygon are distinct whole numbers.

   A. 24 B. 25 C. 26 D. 27

5. There are two perpendicular chords of a circle such that one of the chord was subdivided into two segments of 3 units and 4 units while the other was subdivided into two segments of 6 units and 2 units. What is the diameter of the given circle?

   A. $2\sqrt{15}$ B. $3\sqrt{7}$ C. 8 D. $\sqrt{65}$

6. Below are four different groups of interior angle of $\triangle ABC$, which of them cannot be subdivided into three smaller isosceles triangles of equal legs?

   A. $(50^\circ,60^\circ,70^\circ)$ B. $(50^\circ,50^\circ,80^\circ)$ C. $(45^\circ,45^\circ,90^\circ)$ D. $(20^\circ,20^\circ,140^\circ)$

7. In a convex quadrilateral *ABCD*, $\angle ABC = 30^\circ$, $\angle BCD = 60^\circ$, $BC = 8$, $CD = 1$ and $S_{ABCD}=\frac{13\sqrt{3}}{2}$. What is the length of *AB*?

   A. $\sqrt{3}$ B. $2\sqrt{3}$ C. $3\sqrt{3}$ D. $4\sqrt{3}$

8. Let three real numbers *a*, *b*, *c* satisfy $a+b+c=a^2+b^2+c^2=2$. Determine the numerical value of $\frac{(1-a)^2}{bc}+\frac{(1-b)^2}{ca}+\frac{(1-c)^2}{ab}$.

   A. 3 B. −3 C. 1 D. Undetermined

B. Fill in the blank.（5 points each, a total of 40 points）

9. Let *a*, *b*, *c*, *d* be four distinct real numbers(two numbers are not equal to two numbers) that satisfy $(a+c)(a+d) = 1 = (b+c)(b+d)$. Determine the value of $(a+c)(b+c)$.

10. Give one prime factors of $1+2^{21}+4^{21}$.

11. If the two bases of a trapezoid are 3 units and 4 units, one line segment parallel to two bases and subdivided the trapezoid into two smaller trapezoids of equal area, then what is the length of that segment?

12. If real numbers *a*、*b* satisfy $\sqrt{a^2-2a+1}+\sqrt{36-12a+a^2}=10-|b+3|-|b-2|$, then what is the maximum value of a^2+b^2?

13. Let [*a*] denote the greatest integer not exceeding *a*. For example, [4.1] = 4，[-7.2] = - 8. Determine the rational number *x* (express the answer as improper fraction) that satisfy the equation $x+\frac{2013}{x}=[x]+\frac{2013}{[x]}$.

14. Let $f(x)$, $g(x)$ be two second degree function with each of their leading term is 1. If the four roots of equation $f(g(x))=0$ are 2010, 2011, 2012 and 2013; while the solutions of $g(f(x))=0$ are −2010, −2011, −2012 and −2013, then what is the product of the minimum value of each function?

15. Procedure of performing a ***bubble sort*** operation to a series $a_1, a_2, \cdots, a_n$ is as follows: First, compare the size of a1 from the first and a2 from the second terms in the given series. If $a_1 > a_2$, then swap the two positions in the series (that is, exchange their value), or remain unchanged if they are the equal; then compare the size of second and third term in the series, follow the same rule, that is; if $a_2 > a_3$, then swap, or else remain unchanged if they are equal; then compare the size of the third and fourth and so on, $\cdots$, according to the same rules, when reaching the last two terms, after compare the size of a_{n-1}, a_n and after the end of swapping the location, then stop. Now randomly arrange $1, 2, 3, \cdots, 2013$ as a series and perform the above described "Bubble Sort" technique to this series of 2013 terms. What is the probability that after bubble sort the number in the 10th term is the number in the 5th term?

16. In a given isosceles $\triangle ABC$, $\angle A = 100^\circ$, $AB = AC$ with P an interior point of this $\triangle$ such that $PB = AB$ and $\angle ABP = 2\angle ACP$. What is the size of $\angle APB$?

C. Problem Solving. (10 points each, a total of 20 points. Show your brief solution on the space below each question)

17. Let *BD* and *CE* be the angle bisectors of $\triangle ABC$. Construct the angle bisectors of $\angle ABD$ and $\angle ACE$ at points *B* and *C;* respectively intersect at point *P*, *O* is the circum-center of acute $\triangle ABC$. If *BPOC* is a cyclic quadrilateral and *PB* = *PC*, then what is the measured degree of three interior angle of $\triangle ABC$?

18. The triplet (A, B, C) can be transformed with the following two steps:

Transformation 1: Three numbers can be rearranged arbitrarily

Transformation 2: Convert (A, B, C) into $(2B + 2C - A, B, C)$;

Assume that the initial state of a given triplet is $(-1, 0, 1)$.

(a) Can we perform a finite number of steps of transformation to obtain a triplet as (2012, 2013, 2014). Explain.

(b) Can we perform a finite number of steps of transformation to obtain a triplet such as (2009, 2010, 2011). Explain.

(c) Determine all possible values of x so that there is a finite number of steps of transformation to obtain a triplet as (1, 2024, *x*). Explain